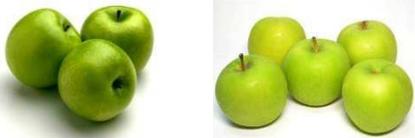




Progression in the teaching of counting in EYFS

<p>Pre-counting</p> <p>The key focus in pre-counting is an understanding of the concepts more, less and the same and an appreciation of how these are related. Children at this stage develop these concepts by comparison and no counting is involved.</p>	<p>Ordering</p> <p>Count by reciting the number names in order forwards and backwards from any starting point.</p>	<p>One to one correspondence</p> <p>One number word has to be matched to each and every object. Lack of coordination is a source of potential error – it helps if children move the objects as they count, use large rhythmic movements, or clap as they count.</p>	<p>Cardinality (Knowing the final number counted is the total number of objects)</p> <p>Count out a number of objects from a larger collection. Know the number they stop counting at will give the total number of objects.</p>
<p>Pre-counting ideas</p> <p>Provide children with opportunities to sort groups of objects explicitly using the language of more and less.</p>  <p>Which group of apples has the most? Which group of apples has the least?</p>	<p>Ordering ideas</p> <p>Provide children with opportunities to count orally on a daily basis. Practise counting so that children are able to understand number order and can hear the rhythm and pattern. Use a drum or clap to keep the beat.</p> 	<p>One to one correspondence ideas</p> <p>Play counting games together moving along a track, play games involving amounts such as knocking down skittles.</p> <p>Use traditional counting songs throughout the day ensuring children have the visual/kinaesthetic resources eg. 5 little ducks, 10 green bottles</p> 	<p>Cardinal counting ideas</p>  <p>How many bananas are in my fruit bowl? Allow children to physically handle the fruit.</p> <p>Provide children with objects to point to and move as they count and say the numbers.</p>



Progression in the teaching of counting in EYFS

Subitising (recognise small numbers without counting them)

Children need to recognise small amounts without counting them eg. dot patterns on dice, dots on tens frames, dominoes and playing cards as well as small groups of randomly arranged shapes stuck on cards.

Abstraction

You can count anything – visible objects, hidden objects, imaginary objects, sounds etc. Children find it harder to count things they cannot move (because the objects are fixed), touch (they are at a distance), see, that move around. Children also find it difficult to count a mix of different objects, or similar objects of very different sizes.

Conservation of number

Ultimately children need to realise that when objects are rearranged the number of them stays the same.

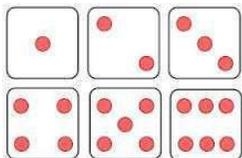
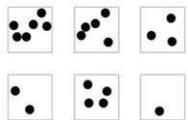
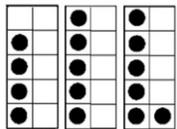
End of year counting expectations

- count reliably to 20
- count reliably up to 10 everyday objects
- estimate a number of objects then check by counting
- use ordinal numbers in context eg first, second, third
- count in twos, fives and tens
- order numbers 1-20
- say 1 more/ 1 less than a given number to 20



Subitising ideas

Provide children with opportunities to count by recognising amounts.



Abstraction ideas



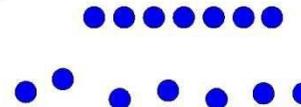
How many pigs are in this picture?

Provide children with a variety of objects to count.



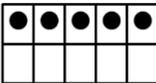
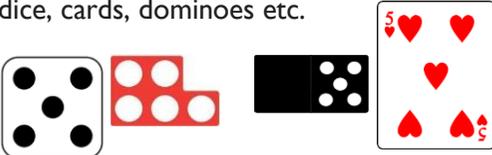
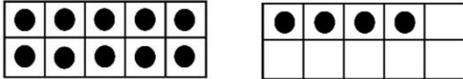
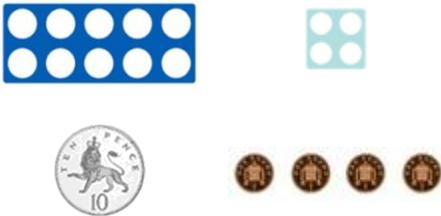
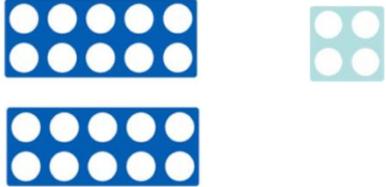
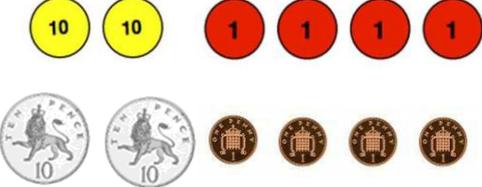
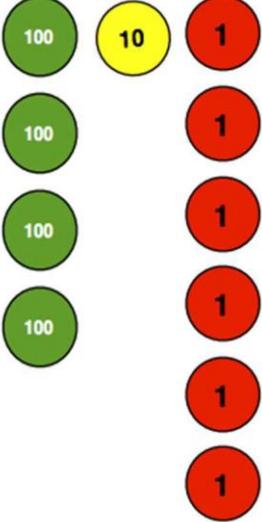
Conservation of Number

- The amount is "seven" and doesn't change.



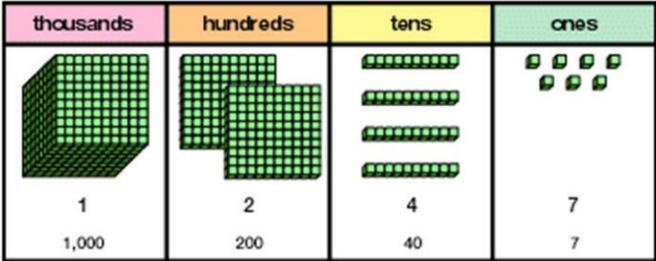
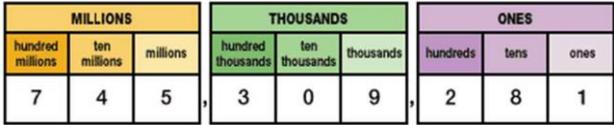
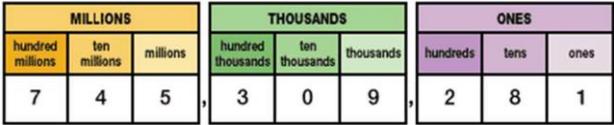


Progression in the teaching of place value

EYFS	Stage 1	Stage 2	Stage 3						
<p align="center">Understanding ten</p>	<p align="center">Understanding numbers up to 20</p>	<p align="center">Understanding numbers up to one hundred</p>	<p align="center">Understanding numbers up to one thousand</p>						
<p>A TENS FRAME is a simple maths tool that helps children:</p> <ul style="list-style-type: none"> • Keep track of counting • See number relationships • Learn addition to 10 • Understand place value <p>Use tens frames flash cards daily to ensure children recognise amounts.</p> <p>Use empty tens frames to fill with counters to enable children to understand number relationships.</p> <p>Either fill the tens frame in pairs or in rows. In rows shows 5 as a benchmark. Children can easily see more than 5 or less.</p>  <p>Setting the counters in pairs, naturally allows the children to see addition concepts.</p> <p>Include other visual images such as dice, cards, dominoes etc.</p> 	<p>'Ten' is the building block of our Base 10 numeration system. Young children can usually 'read' two-digit numbers long before they understand the effect the placement of each digit has on its numerical value. A child might be able to correctly read 62 as sixty- two and 26 as twenty-six, and even know which number is larger, without understanding why the numbers are of differing values.</p> <p>Ten-frames can provide a first step into understanding two-digit numbers simply by the introduction of a second frame. Placing the second frame to the right of the first frame, and later introducing numeral cards, will further assist the development of place- value understanding.</p>   	<p>Continue developing place value through the use of tens frames.</p>    	<p>Continue developing place value through the use of manipulatives.</p>   <p>Use Dienes blocks and place value charts.</p> <table border="1" data-bbox="1657 1252 2116 1508"> <thead> <tr> <th>Hundreds</th> <th>Tens</th> <th>Ones</th> </tr> </thead> <tbody> <tr> <td></td> <td></td> <td></td> </tr> </tbody> </table>	Hundreds	Tens	Ones			
Hundreds	Tens	Ones							



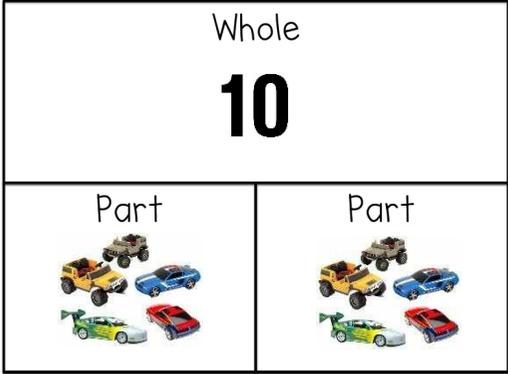
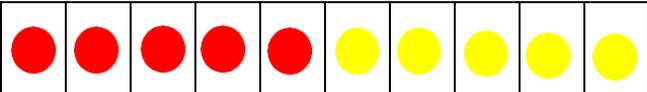
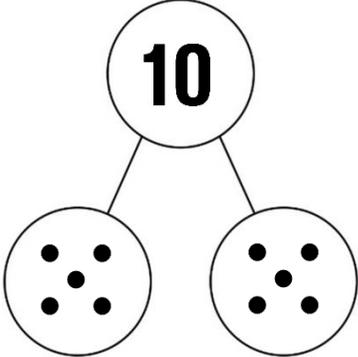
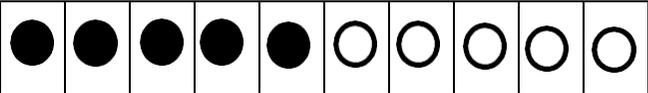
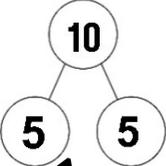
Progression in the teaching of place value

Stage 4	Stage 5	Stage 6																																																																		
<p>Understanding numbers up to ten thousand</p>	<p>Understanding numbers up to one million including decimals</p>	<p>Understanding numbers beyond one million including decimals</p>																																																																		
<p>Continue developing place value through the use of manipulatives.</p> <ul style="list-style-type: none"> Place value arrow cards Place value counters Dienes blocks Place value charts  <table border="1" data-bbox="85 770 741 1031"> <thead> <tr> <th>thousands</th> <th>hundreds</th> <th>tens</th> <th>ones</th> </tr> </thead> <tbody> <tr> <td></td> <td></td> <td></td> <td></td> </tr> <tr> <td>1 1,000</td> <td>2 200</td> <td>4 40</td> <td>7 7</td> </tr> </tbody> </table>	thousands	hundreds	tens	ones					1 1,000	2 200	4 40	7 7	<p>Continue developing place value through the use of manipulatives.</p> <ul style="list-style-type: none"> Place value arrow cards Place value counters (including decimal counters) Dienes blocks Place value charts  <table border="1" data-bbox="815 770 1429 895"> <thead> <tr> <th colspan="3">MILLIONS</th> <th colspan="3">THOUSANDS</th> <th colspan="3">ONES</th> </tr> <tr> <th>hundred millions</th> <th>ten millions</th> <th>millions</th> <th>hundred thousands</th> <th>ten thousands</th> <th>thousands</th> <th>hundreds</th> <th>tens</th> <th>ones</th> </tr> </thead> <tbody> <tr> <td>7</td> <td>4</td> <td>5</td> <td>3</td> <td>0</td> <td>9</td> <td>2</td> <td>8</td> <td>1</td> </tr> </tbody> </table>	MILLIONS			THOUSANDS			ONES			hundred millions	ten millions	millions	hundred thousands	ten thousands	thousands	hundreds	tens	ones	7	4	5	3	0	9	2	8	1	<p>Continue developing place value through the use of manipulatives.</p> <ul style="list-style-type: none"> Place value arrow cards Place value counters (including decimals counters) Dienes blocks Place value charts  <table border="1" data-bbox="1518 770 2132 895"> <thead> <tr> <th colspan="3">MILLIONS</th> <th colspan="3">THOUSANDS</th> <th colspan="3">ONES</th> </tr> <tr> <th>hundred millions</th> <th>ten millions</th> <th>millions</th> <th>hundred thousands</th> <th>ten thousands</th> <th>thousands</th> <th>hundreds</th> <th>tens</th> <th>ones</th> </tr> </thead> <tbody> <tr> <td>7</td> <td>4</td> <td>5</td> <td>3</td> <td>0</td> <td>9</td> <td>2</td> <td>8</td> <td>1</td> </tr> </tbody> </table>	MILLIONS			THOUSANDS			ONES			hundred millions	ten millions	millions	hundred thousands	ten thousands	thousands	hundreds	tens	ones	7	4	5	3	0	9	2	8	1
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Progression in the teaching of calculations

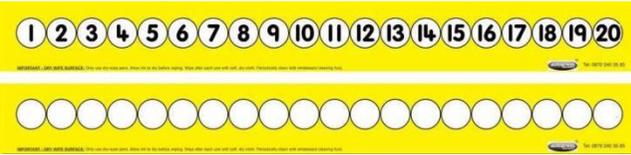
Addition

Objective and strategies	Concrete	Pictorial	Abstract
<p>Combine two parts to make a whole model.</p> <p>Part-part-whole model</p> <p>Teach the children that the cubes/counters represent the real-life objects.</p> <p>Use cubes to add two numbers together as a group or in a bar.</p>	<p>Part, Part, Whole Mat</p>    	   <p>Part + Part = Whole</p> <p>Whole - Part = Part</p>	 <p>Use the part-part whole diagram as shown above to move into the abstract.</p> <p>$5 + 5 = 10$</p> <p>$10 = 5 + 5$</p> <p>Use the term 'number equation'.</p>

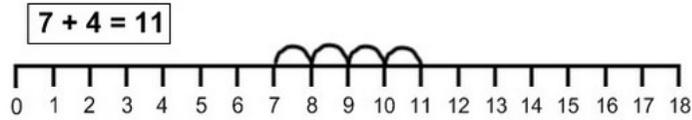
Start at the larger number and count on



Use counters on a number track to count on.



Start with the larger number on the bead string then count on 1 by 1 to find the answer.



Start at the larger number on the number line and count on in ones or in one jump to find the answer.

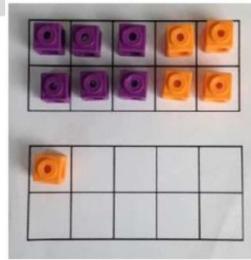
$$4 + 7 = 11$$

Place the larger number in your head and count on the smaller number to find your answer.

Regrouping to make 10.



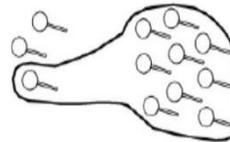
$$6 + 5 = 11$$



$$4 + 7 + 6 = 17$$

Put 4 and 6 together to make 10. Add on 7.

Start with the bigger number and use the smaller number to make 10.

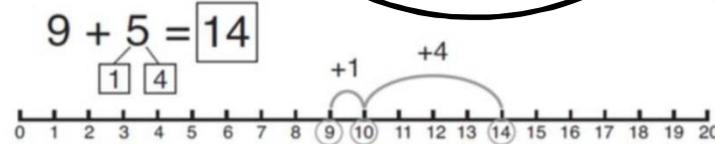


$$3 + 9 =$$

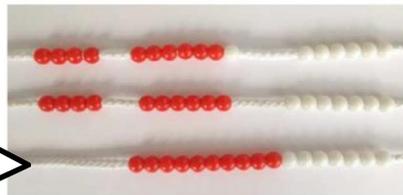
Use pictures or a number line. Regroup or partition the smaller number to make 10.

$$7 + 4 = 11$$

If I am at seven, how many more do I need to make 10. How many more do I add on now?



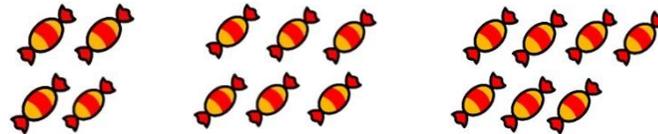
Adding three single digits.



Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit.

Encourage children to use known facts.

Add together three groups of objects. Draw a picture to recombine the groups to make 10.



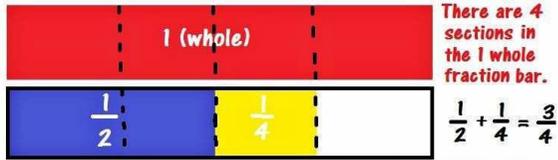
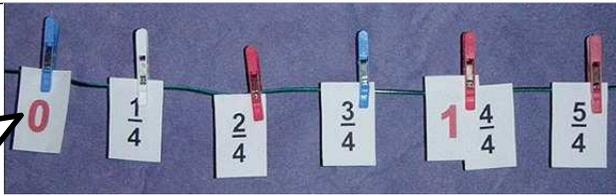
$$4 + 6 + 7 = 17$$

$$\begin{aligned} 4 + 7 + 6 &= 10 + 7 \\ &= 17 \end{aligned}$$

Combine the two numbers that make 10 and then add on the remainder.

Add fractions

Count in fraction steps using real objects and a number line.



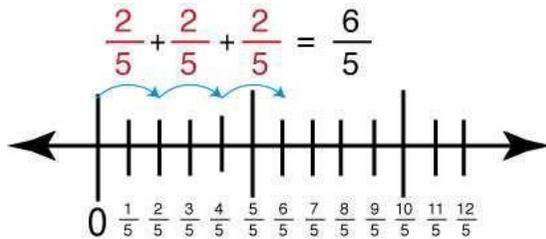
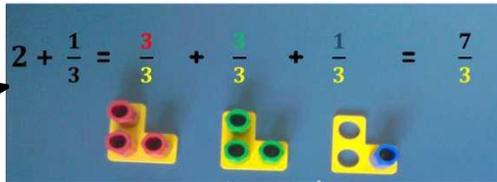
When I add the $\frac{1}{2}$ with the $\frac{1}{4}$ it matches the same space as three sections in the 'benchmark' one whole fraction bar.

LCD = least common denominator

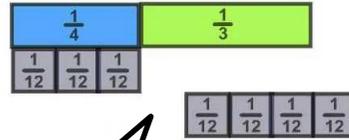
Find the LCD of 2 and 4 so you have a common denominator. Convert then add the numerators.

$$\begin{array}{r} \frac{1}{2} \times 2 = \frac{2}{4} \\ + \frac{1}{4} \times 1 = \frac{1}{4} \\ \hline \frac{3}{4} \end{array}$$

Use Numicon to add fractions.



$$\frac{1}{4} + \frac{1}{3}$$



Use the bar model to add fractions.

$$\frac{1}{4} + \frac{1}{3} =$$

$$\frac{1 \times 3}{4 \times 3} + \frac{1 \times 4}{3 \times 4}$$

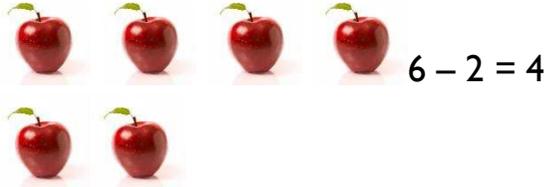
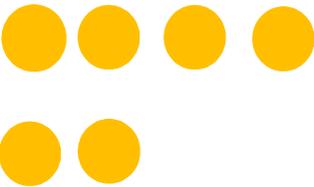
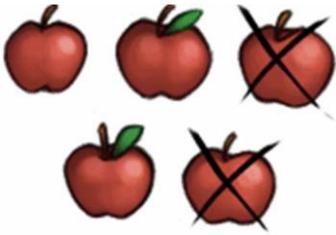
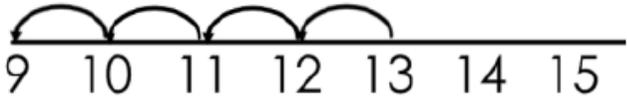
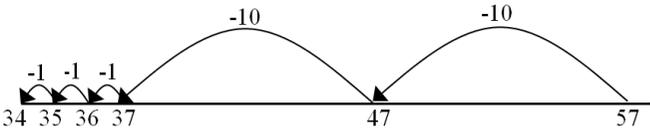
$$\frac{3}{12} + \frac{4}{12} = \frac{7}{12}$$

Multiply to find the common denominator.



Progression in Calculations

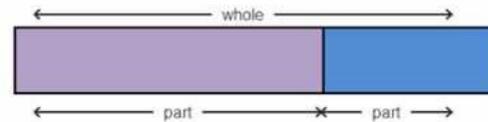
Subtraction

Objective and strategies	Concrete	Pictorial	Abstract
<p>Subtracting ones</p>	<p>Use real-life physical objects, counters, cubes etc. to show how objects can be taken away.</p>  	<p>Cross out drawn objects to show what has been taken away.</p> 	<p>$4 = 6 - 2$</p> <p>$18 - 3 = 15$</p> <p>$8 - 2 = 6$</p>
<p>Counting back</p> <div data-bbox="67 1236 488 1548" style="border: 1px solid black; border-radius: 50%; padding: 10px; width: fit-content;"> <p>Use counters and move them away from the group whilst counting backwards.</p> </div>	<p>Make the larger number in the subtraction calculation. Move the beads along the bead string whilst counting backwards in ones.</p>  	<p>Count back on a number line or number track</p>  <p>Start at the bigger number and count back the smaller number showing the jumps on the number line.</p> 	<p>Put 13 in your head, count back 4. What number are you at? Use your fingers to help.</p> <div data-bbox="1780 1168 2161 1471" style="border: 1px solid black; border-radius: 50%; padding: 10px; width: fit-content;"> <p>Children will need regular practice counting backwards.</p> </div>

Use cubes to subtract a number from the bar.



Use the bar



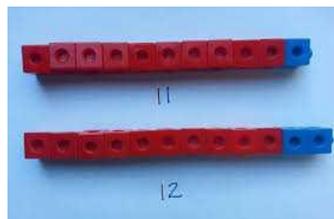
$$\text{Part} + \text{Part} = \text{Whole}$$

$$\text{Whole} - \text{Part} = \text{Part}$$

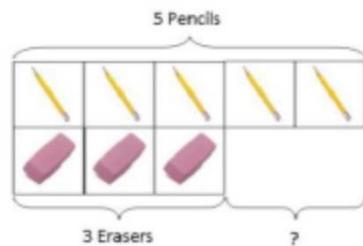
Find the difference

Compare amounts and objects to find the difference.

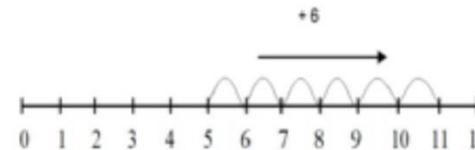
Use cubes to build towers or make bars to find the difference.



Use basic bar models with items to find the difference.

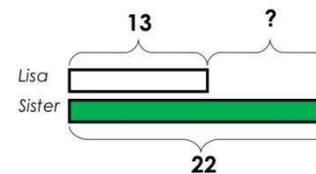


Count on to find the difference.



Comparison Bar Models

Lisa is 13 years old. Her sister is 22 years old. Find the difference in age between them.

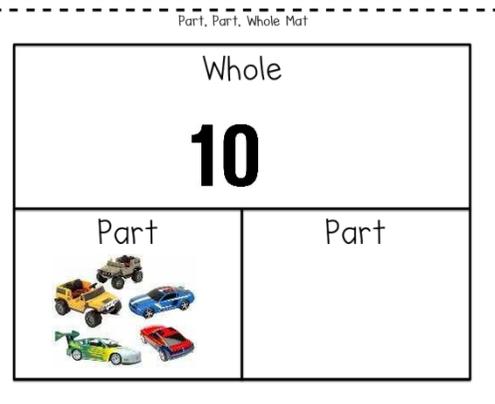


Hannah has 23 pencils, Helen has 15 pencils. Find the difference between the number of pencils.

Draw bars to find the difference between two numbers.

Part-part-whole model

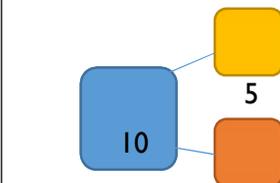
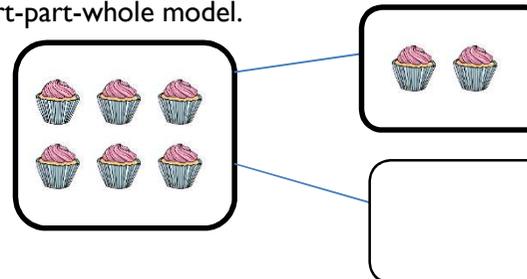
Link to addition-use the part whole model to help explain the inverse.



If 10 is the whole and 5 is one of the parts. What is the other part?

$$10 - 5 = \quad \text{or} \quad 10 - ? = 5$$

Use a pictorial representation of objects to show the part-part-whole model.

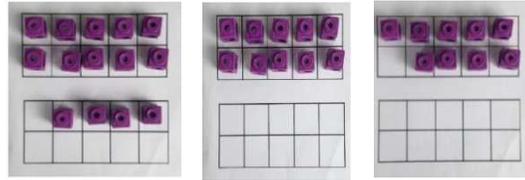


$$10 - 5 = 5 \quad \text{or} \quad 5 = 10 - ?$$

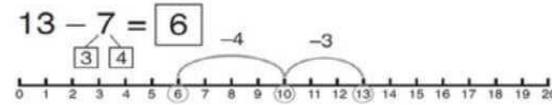
Move to using numbers with the part-part-whole model.

Make 10

$14 - 5 =$



Make 14 on the ten frame. Take away the four first to make 10 and then take away one more so you have taken away 5. You are left with the answer of 9.



Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. You have reached your answer.

$16 - 8 =$

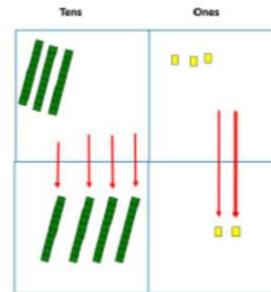
How many do we take off to reach the next 10?

How many do we have left to take off?

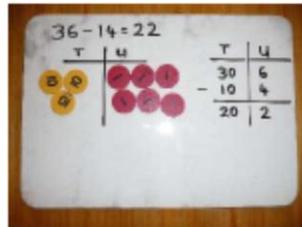
Column method without regrouping

$75 - 42 =$

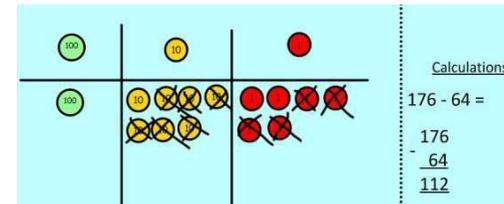
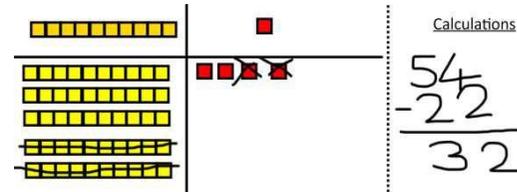
Use Dienes to make the bigger then take the smaller number away.



Show how you partition numbers to subtract. Again make the larger number first.



Draw the Dienes or place value counters alongside the written calculation to help to show working.



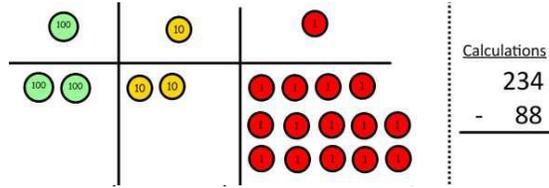
This will lead to a clear written column subtraction.

$$47 - 24 = 23$$

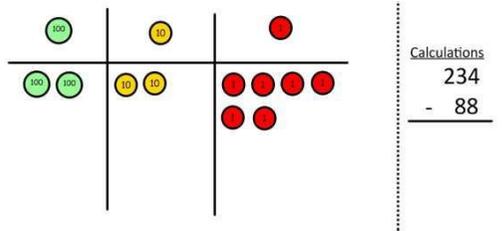
$$\begin{array}{r} 40 + 7 \\ - 20 + 4 \\ \hline 20 + 3 \end{array}$$

Column method with regrouping

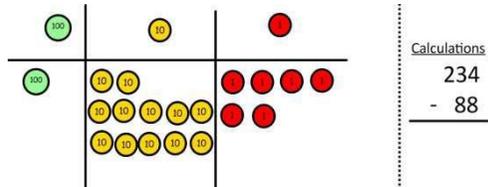
Make the larger number with the Dienes or place value counters. Start with the ones. Can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones.



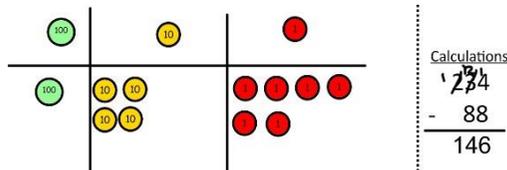
Now I can subtract my ones.



Now look at the tens. Can I take away 8 tens easily? I need to exchange one hundred for ten tens.

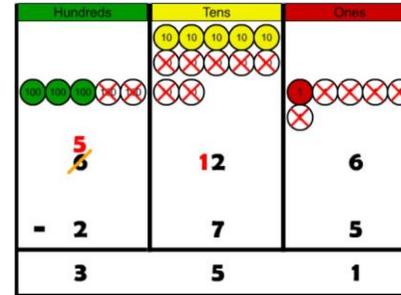


Now I can take away eight tens and complete my subtraction

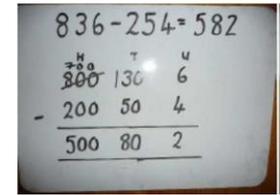


Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount.

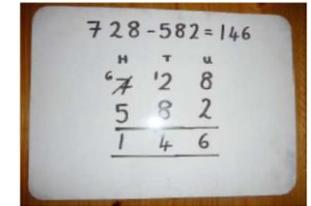
Draw the counters onto a place value grid and show what has been taken away by crossing the counters out as well as clearly showing the exchanges made.



When confident, children can find their own way to record the exchange/regrouping.

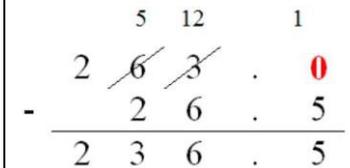


Children can start their formal written method by partitioning the number into clear place value columns.



Moving forward the children use a more compact method.

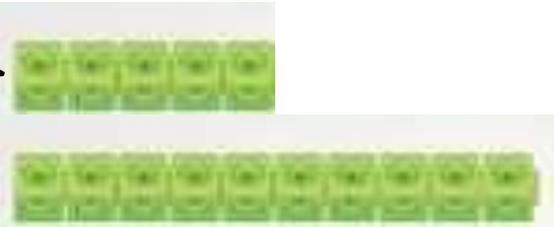
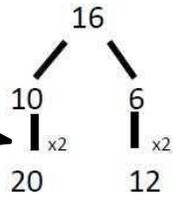
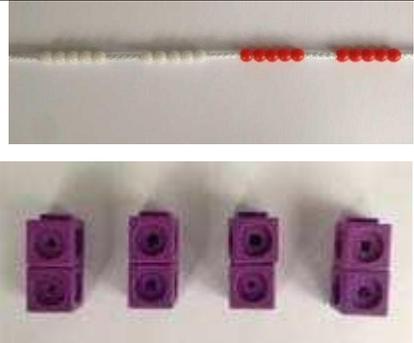
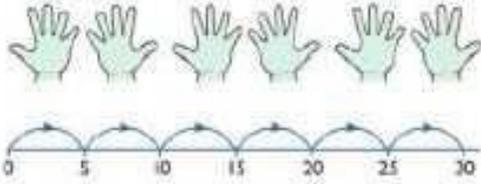
This will lead to an understanding of subtracting any number including decimals.



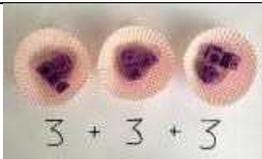


Progression in Calculations Policy

Multiplication

Objective and strategies	Concrete	Pictorial	Abstract
<p>Doubling</p> <p>Double five is ten.</p>	<p>Use practical activities to show how to double a number.</p>  <p>$5 \times 2 = 10$</p>	<p>Draw pictures to show how to double a number.</p> <p>Double 4 is 8</p>  <p>Double the 10 then double the 6.</p>	<p>Double 16</p>  <p>Partition a number and then double each part before recombining it back together.</p> <p>Count in multiples of a number aloud.</p>
<p>Counting in multiples</p>	 <p>Count in multiples supported by concrete objects in equal groups.</p>	 <p>Use a number line or pictures to continue support in counting in multiples.</p>	<p>Write sequences with multiples of numbers.</p> <p>2, 4, 6, 8, 10</p> <p>5, 10, 15, 20, 25, 30</p>

Repeated addition

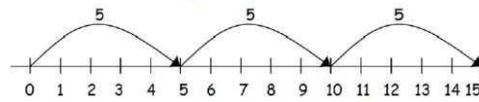


Use different objects to add equal groups.

There are 3 plates. Each plate has 2 star biscuits on. How many biscuits are there?



2 add 2 add 2 equals 6



$$5 + 5 + 5 = 15$$

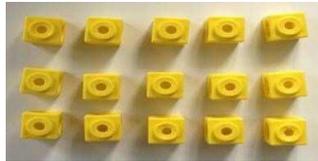
Write addition sentences to describe objects and pictures.



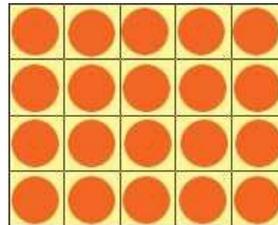
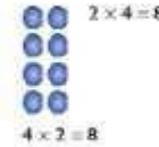
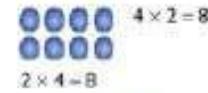
$$2 + 2 + 2 + 2 + 2 = 10$$

Arrays- showing commutative multiplication

Create arrays using counters/ cubes to show multiplication sentences.



Draw arrays in different rotations to find **commutative** multiplication sentences.



Link arrays to area of rectangles.

Use an array to write multiplication sentences and reinforce repeated addition.



$$5 + 5 + 5 = 15$$

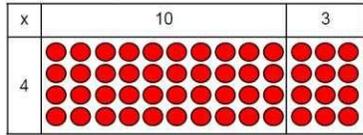
$$3 + 3 + 3 + 3 + 3 = 15$$

$$5 \times 3 = 15$$

$$3 \times 5 = 15$$

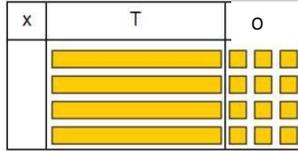
Grid Method

Show the link with arrays to first introduce the grid method.



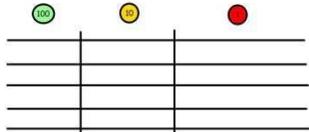
4 rows of 10
4 rows of 3

Use Dienes to move towards a more compact method.



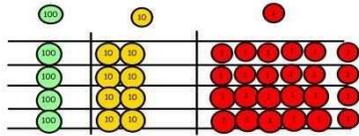
4 rows of 13

Use place value counters to show finding groups of a number eg. multiplying by 4 so we need 4 rows.



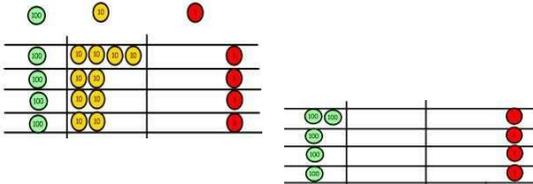
Calculations
 4×126

Fill each row with 126.



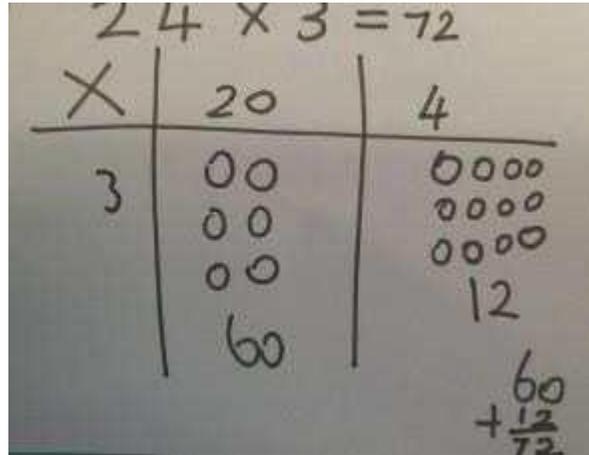
Calculations
 4×126

Add up each column, starting with the ones making any exchanges needed.



Children can represent the work they have done with place value counters in a way that they understand.

They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below.



Start with multiplying by one digit numbers and showing the clear addition alongside the grid.

X	30	5
7	210	35

$$210 + 35 = 245$$

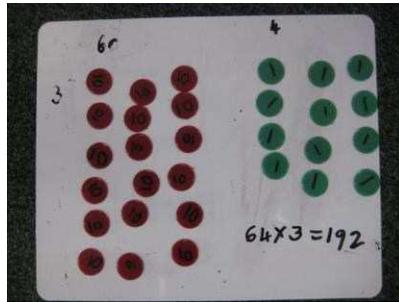
Moving forward, multiply by a 2 digit number showing the different rows within the grid method.

	10	8
10	100	80
3	30	24

X	1000	300	40	2
10	10000	3000	400	20
8	8000	2400	320	16

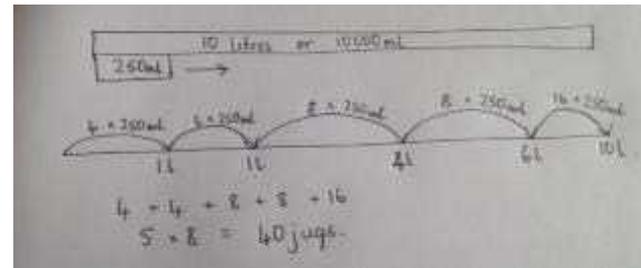
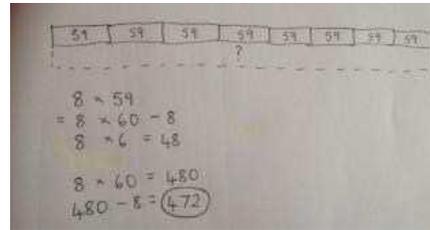
Column multiplication

Children can continue to be supported by place value counters at the stage of multiplication.



It is important at this stage that they always multiply the ones first and note down their answer followed by the tens which they note below.

Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.



Start with long multiplication, reminding the children about lining up their numbers clearly in columns.

If it helps, children can write out what they are solving next to their answer.

$$\begin{array}{r}
 32 \\
 \times 24 \\
 \hline
 8 \quad (4 \times 2) \\
 120 \quad (4 \times 30) \\
 40 \quad (20 \times 2) \\
 600 \quad (20 \times 30) \\
 \hline
 768
 \end{array}$$

$$\begin{array}{r}
 7 \ 4 \\
 \times 6 \ 3 \\
 \hline
 1 \ 2 \\
 2 \ 1 \ 0 \\
 2 \ 4 \ 0 \\
 + 4 \ 2 \ 0 \ 0 \\
 \hline
 4 \ 6 \ 6 \ 2
 \end{array}$$

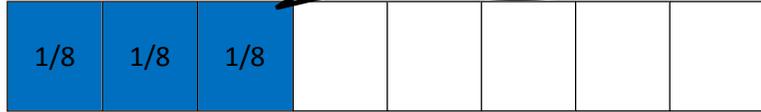
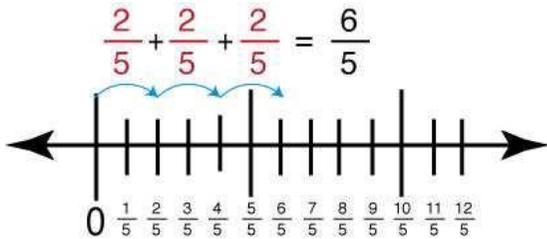
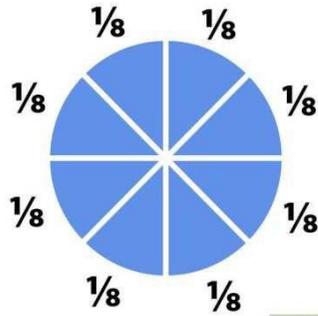
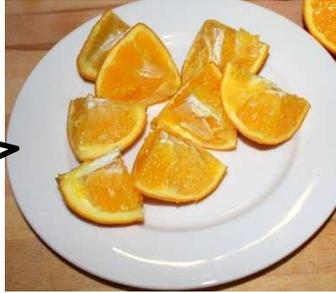
This moves to the more compact method.

$$\begin{array}{r}
 327 \\
 \times 53 \\
 \hline
 981 \quad \leftarrow 327 \times 3 \\
 16350 \quad \leftarrow 327 \times 50 \\
 \hline
 17331
 \end{array}$$

Multiplication of fractions

Count in fraction steps (repeated addition)

What would three lots of one eighth be?



Three times one eighth.

$\frac{10}{8} = 1 \frac{2}{8}$
$\frac{9}{8} = 1 \frac{1}{8}$
$\frac{8}{8} = 1$
$\frac{7}{8}$
$\frac{6}{8}$
$\frac{5}{8}$
$\frac{4}{8} = \frac{1}{2}$
$\frac{3}{8}$
$\frac{2}{8} = \frac{1}{4}$
$\frac{1}{8}$

$3 \times \frac{1}{8} =$

$\frac{3}{1} \times \frac{1}{8} = \frac{3}{8}$

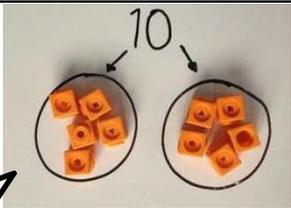
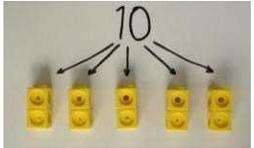
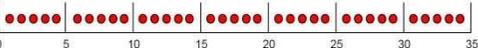
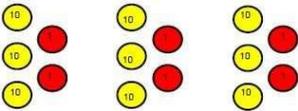
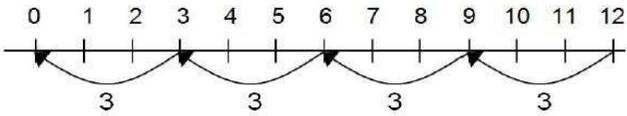
Multiply the numerators together then multiply the denominators.



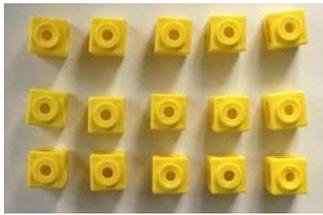
Progression in Calculations Policy

Division

It is important to make links with fractions

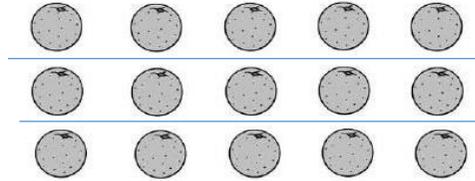
Objective and strategies	Concrete	Pictorial	Abstract
<p>Sharing objects into groups</p> <p>If we are dividing by two we are finding one half.</p>	 <p>I have 10 cubes; can you share them equally into 2 groups?</p>	<p>Children use pictures or shapes to share quantities.</p>  <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> $8 \div 2 = 4$ </div>	<p>One half of 14 is 7 $\frac{1}{2}$ of 14 = 7 $14 \div 2 = 7$</p> <p>Share 9 cakes between three people.</p> $9 \div 3 = 3$
<p>Division as grouping</p> <p>If we are dividing by three we are finding one third.</p>	<p>Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.</p>   $96 \div 3 = 32$ 	<p>Use a number line to show jumps in groups. The number of jumps equals the number of groups.</p>  <p>Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.</p>  $20 \div 5 = ?$ $5 \times ? = 20$	<p>$28 \div 7 = 4$</p> <p>Divide 28 into 7 groups. How many are in each group?</p>

Division within arrays



Link division to multiplication by creating an array and thinking about the number sentences that can be created.

Eg $15 \div 3 = 5$ $5 \times 3 = 15$
 $15 \div 5 = 3$ $3 \times 5 = 15$



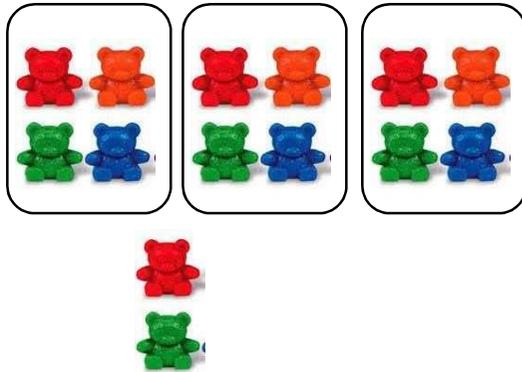
Draw an array and use lines to split the array into groups to make multiplication and division sentences.

Find the inverse of multiplication and division sentences by creating four linking number sentences.

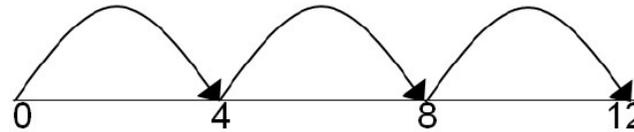
$7 \times 4 = 28$
 $4 \times 7 = 28$
 $28 \div 7 = 4$
 $28 \div 4 = 7$

Division with a remainder

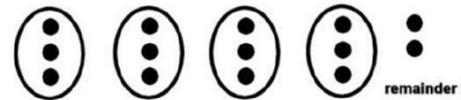
$14 \div 3 =$
 Divide objects between groups and see how much is left over



Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.



Draw dots and group them to divide an amount and clearly show a remainder.



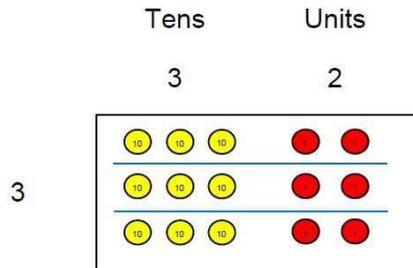
Complete written divisions and show the remainder using r.

$29 \div 8 = 3 \text{ REMAINDER } 5$
 ↑ ↑ ↑ ↑
 dividend divisor quotient remainder

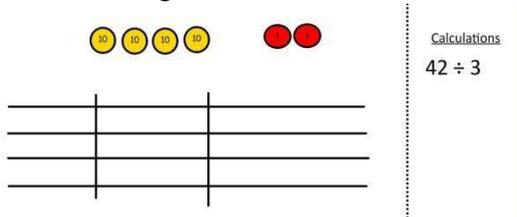
Short division

Find one third of 96.

$$96 \div 3 =$$

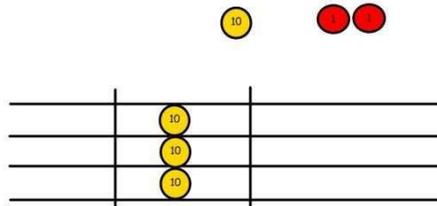


Use place value counters to divide using the bus stop method alongside

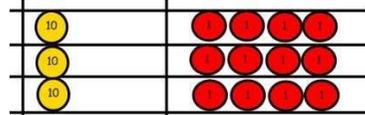


$$42 \div 3 =$$

Start with the biggest place value; share 40 into three groups. Put 1 ten in each group then 1 ten left over.

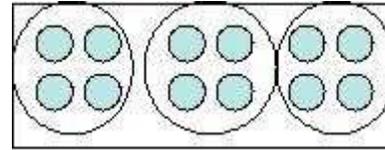


Exchange this ten for ten ones and then share the ones equally among the groups.



Look how much is in 1 group so the answer is 14.

Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.



Encourage them to move towards counting in multiples to divide more efficiently.

Begin with divisions that divide equally with no remainder.

$$\begin{array}{r} 218 \\ 3 \overline{) 872} \\ \underline{6} \\ 27 \\ \underline{27} \\ 0 \end{array}$$

Move onto divisions with a remainder.

$$\begin{array}{r} 86 \text{ r } 2 \\ 5 \overline{) 432} \\ \underline{4} \\ 3 \\ \underline{3} \\ 2 \end{array}$$

Finally move into decimal places to divide the total accurately.

$$\begin{array}{r} 14.6 \\ 35 \overline{) 511.0} \\ \underline{35} \\ 16 \\ \underline{15} \\ 1 \\ \underline{10} \\ 10 \\ \underline{10} \\ 0 \end{array}$$

Long division

Divide by single digit then progress to dividing by two digit number

$$\begin{array}{r}
 86 \text{ r}2 \\
 5 \overline{) 432} \\
 \underline{200} \quad (40 \times 5) \\
 232 \\
 \underline{200} \quad (40 \times 5) \\
 32 \\
 \underline{30} \quad (6 \times 5) \\
 2
 \end{array}$$

$$\begin{array}{r}
 13 \overline{) 1937} \\
 - 1300 \quad 13 \times 100 \\
 \hline
 637 \\
 - 520 \quad 13 \times 40 \\
 \hline
 117 \\
 - 117 \quad 13 \times 9 \\
 \hline
 0
 \end{array}$$

Division of fractions

$\frac{1}{2} \div 3 =$

Half of the pizza divided into three equal parts.



$\frac{1}{2} \div 3 =$



Half of the bar divided into three equal parts.

$\frac{1}{2} \div 3 =$

$\frac{1}{2} \div \frac{3}{1} =$

$\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$



Reasoning, Greater Depth and Mastery



What do we mean by mastery?

The essential idea behind mastery is that *all children* need a *deep* understanding of the mathematics they are learning so that:

- future mathematical learning is built on solid foundations which do not need to be re-taught;
- there is no need for separate catch-up programmes due to some children falling behind;
- children who, under other teaching approaches, can often fall a long way behind, are better able to keep up with their peers, so that gaps in attainment are narrowed whilst the attainment of all is raised.

There are generally four ways in which the term mastery is being used in the current debate about raising standards in mathematics:

- 1. A mastery approach:** a set of principles and beliefs. This includes a belief that all pupils are capable of understanding and doing mathematics, given sufficient time. Pupils are neither 'born with the maths gene' nor 'just no good at maths'. With good teaching, appropriate resources, effort and a 'can do' attitude all children can achieve in and enjoy mathematics.
- 2. A mastery curriculum:** one set of mathematical concepts and big ideas for all. All pupils need access to these concepts and ideas and to the rich connections between them. Mathematics is mathematics and the key ideas and building blocks are important for everyone.
- 3. Teaching for mastery:** a set of pedagogic practices that keep the class working together on the same topic, whilst at the same time addressing the need for all pupils to master the curriculum and for some to gain greater depth of proficiency and understanding. Challenge is provided by going depth and sufficient practice to embed learning.

4. Achieving mastery of particular topics and areas of mathematics. Mastery is not just being able to memorise key facts and procedures and answer test questions accurately and quickly. It involves knowing 'why' as well as knowing 'that' and knowing 'how'. It means being able to use one's knowledge appropriately, flexibly and creatively and to apply it in new and unfamiliar situations.

Mastery of the curriculum requires that all pupils:

- use mathematical concepts, facts and procedures appropriately, flexibly and fluently;
- recall key number facts with speed and accuracy and use them to calculate and work out unknown facts;
- have sufficient depth of knowledge and understanding to reason and explain mathematical concepts and procedures and use them to solve a variety of problems.

Odd one out	Would you rather have ...	Find the mistake.	What is the same and what is different?
True or false?	Here is the	Always, sometimes, never	Give me a silly answer to this problem. What makes it silly?
Tell me about this...	Prove/disprove this statement.	Convince me that ...	What if....?
Give me a hard and easy example of a calculation you could do with these numbers. Give me a hard and easy example of a five-digit calculation. Give me a hard and easy example of a question you could ask about this graph/pie chart etc.	What do you	How are these linked?	If you know this fact, what else do you know? Eg. If you know: $4 + 6 = 10$ You know: $40 + 60 = 100$ $100 - 40 = 60$ The sum of 6 and 4 is 10. $4000 + 6000 = 10,000$ $100,000 - 60,000 = 40,000$ If it is 6 o'clock now, in 4 hours it will be 10 o'clock.

